RAN-2101001102050001

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M. A. (Part - II) External - Examination April - 2026

Mathematics (Paper - 5001)

Integral Transforms

Time: 3 Hours] **[Total Marks: 100**

સૂચના : / Instructions	Seat No.:
(1) ઉપરોક્ત દર્શાવેલ વિગતો ઉત્તરવહી પર અવશ્ય લખવી. Fill up strictly the above details on your answer book.	
(2) Answer all questions.	
(3) Figures to the right indicates marks.	
(4) Follow usual notations and conventions.	Student's Signature

Q.1 (a) If $L[f(t)] = \bar{f}(s)$, then prove that 07

(i) $\lim\limits_{t\to 0} f(t) = \lim\limits_{s\to\infty} s\ \overline{f(s)}$

(ii) $\lim\limits_{t\to\infty} f(t) = \lim\limits_{s\to 0} s\ \overline{f(s)}$

(b) Define Laplace transform. Use $L^{-1}\left[\frac{s^2-1}{(s^2+1)^2}\right] = tcost$ to find $L^{-1}\left[\frac{9s^2-1}{(9s^2+1)^2}\right]$. 07

(c) Find (i) $L[(1+te^{-3t})^3]$ 06

(ii) $L[t^2 sinat]$

OR

Q.1 (a) If $L[f(t)] = \overline{f(s)}$, then prove that $L[t^n f(t)] = (-1)^n \frac{d^n}{ds^n}\overline{f(s)}$ 07

Hence find $L[tcosat]$.

(b) Find Laplace Transform of $f(t) = \begin{cases} (t-1)^2 & t > 1 \\ 0 & 0 < t < 1 \end{cases}$. 07

(c) Find $L^{-1}\left[\frac{1}{(s^2+a^2)(s^2+b^2)}\right]$. 06

Q.2 (a) Use Laplace transform to solve 07

$$y'' + 25y = 10cos5t\ , y(0) = 2, y'(0) = 0.$$

(b) Use Laplace transform to solve the integral equation 07

$$\int_0^t f(u)(t-u)^{-1/2}du = 1 + t + t^2$$

(c) Using convolution theorem evaluate $L^{-1}\left[\frac{1}{(s+a)(s+b)}\right]$. 06

OR

Q.2 (a) If $L^{-1}[\bar{f}(s)] = f(t)$, then prove that $L^{-1}[\bar{f}(as)] = \frac{1}{a} f(\frac{t}{a})$, where $a > 0$. 07
Find $L^{-1}\left[log\left(\frac{s+3}{s+2}\right)\right]$.

(b) Using Laplace transform, solve the integral equation 07

$$f(t) = t + 2\int_0^t \cos(t-u)\, f(u)du$$

(c) Solve $u_t + xu_x = x$, $x > 0, t > 0$, with the initial and boundary conditions $u(x,0) = 0, for\ x > 0$, $u(0,t) = 0, for\ t > 0$ using Laplace transform. 06

Q.3 (a) Define finite Laplace transform. 07
Find finite Laplace transform of (i) $f(t) = t^2$
(ii) $f(t) = cosat$

(b) (i) Find $L^{-1}\left[\frac{1}{(s-1)\sqrt{s}}\right]$ 07
(ii) Find $L^{-1}\left[\frac{3s+7}{(s^2-2s-3)}\right]$

(c) Find finite cosine transform of $f(x) = \left(1-\frac{x}{\pi}\right)^2$. 06

OR

Q.3 (a) If $f(t)$ is a periodic function with period ω. i.e. $f(t+n\omega) = f(t), (n \in N)$. 07
Then prove that $L[f(t+n\omega)] = \frac{\int_0^\omega e^{-st} f(t)}{1-e^{-s\omega}} dt.$

(b) Using Laplace transform evaluate $\int_0^\infty \left(\frac{e^{-t}-e^{-3t}}{t}\right) dt$ 07

(c) Find finite sine and cosine transform of $f(x) = 1$. 06

Q.4 (a) Use finite Fourier Sine transform to solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ $(0 < x < \pi, t > 0)$, given that (i) $u(0,t) = 0,\ u(\pi,t) = 0$ 07
(ii) $u(x,0) = 2x,\ 0 < x < \pi, t > 0$.

(b) Solve using proper Fourier transform $\frac{\partial u}{\partial t} = 2\frac{\partial^2 u}{\partial x^2}$, 07
subject to boundary conditions
$u(0,t) = 0$, $u(x,0) = e^{-x}$ and $u(x,t)$ is bounded.

(c) Solve the integral equation using Fourier transform $\int_0^\infty f(x)\cos(\lambda x)dx = e^{-\lambda}$ 06

OR

Q.4 (a) Use finite Fourier Cosine transform to solve 07

$\frac{\partial v}{\partial t} = k\frac{\partial^2 v}{\partial x^2}$ $(0 < x < \pi, t > 0)$ with the conditions

$\frac{\partial v}{\partial x} = 0$, for $x = 0; x = \pi$ for $t > 0$

$v = f(x)$, for $t = 0; 0 < x < \pi$

(b) Use Fourier transform to determine the displacement $y(x,t)$ of an infinite string given that string is initially at rest and the initial displacement is $f(x)$ for $-\infty < x < \infty$. Show that solution can be put in the form 07

$y(x,t) = \frac{1}{2}\left[f(x+ct) + f(x-ct)\right]$.

(c) Solve following integral equation using Fourier transform 06

$$\int_0^\infty f(x)\sin(xt)dt = \begin{cases} 1 & ; 0 \le t < 1 \\ 2 & ; 1 \le t < 2 \\ 0 & ; t \ge 2 \end{cases}$$

Q.5 (a) In usual notation prove that: 07

(i) $F_s[f(x)cos(ax)] = \frac{1}{2}[f_s(s+a) + f_s(s-a)]$

(ii) $F_s[f(x)sin(ax)] = \frac{1}{2}[f_c(s-a) - f_c(s+a)]$

(b) If function $f(x)$ satisfies following two conditions: 07

(i) $f(x)$ satisfies Drichlet condition in every open interval $(-l, l)$

(ii) $f(x)$ is absolutely integrable in the interval $(-\infty, \infty)$.i.e. $\int_{-\infty}^{\infty} f(x)dx$ is convergent. Then prove that $f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x)\cos(u(x-t))dudt$

(c) Find finite cosine transform of $f(x) = \begin{cases} 1 & ; 0 \le x \le \frac{\pi}{2} \\ -1 & ; \frac{\pi}{2} \le x \le \pi \end{cases}$ 06

OR

Q.5

(a) Find $f(x)$ if its Fourier sine transform is $\frac{e^{-as}}{s}$. 07

(b) Find finite sine and cosine transform of $f(x) = 2x; 0 < x < 4$. 07

(c) Find the Fourier transform of function $f(x)$, $f(x) = \begin{cases} x & ; if\ |x| \le a \\ 0 & ; if\ |x| > a \end{cases}$. 06
